DIFFERENTIATION

1 A curve has the equation $y = x^2(2 - x)^3$ and passes through the point A (1, 1).

a Find an equation for the tangent to the curve at A.

b Show that the normal to the curve at A passes through the origin.

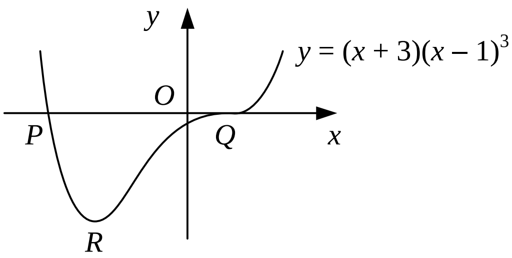
2 A curve has the equation $y = \frac{x}{2x+3}$.

a Find an equation for the tangent to the curve at the point P (−1, −1).

b Find an equation for the normal to the curve at the origin, O.

c Find the coordinates of the point where the tangent to the curve at P meets the normal to the curve at O.

3



The diagram shows the curve with equation $y = (x + 3)(x - 1)^3$ which crosses the x-axis at the points P and Q and has a minimum at the point R.

a Write down the coordinates of P and Q.

b Find the coordinates of R.

4 Given that $y = x\sqrt{4x+1}$,

a show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x+1}{\sqrt{4x+1}}$,

b solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 0$.

5 A curve has the equation $y = \frac{2(x-1)}{x^2+3}$ and crosses the x-axis at the point A.

a Show that the normal to the curve at A has the equation $y = 2 - 2x$.

b Find the coordinates of any stationary points on the curve.

6
$$\mathrm{f}(x) \equiv x^{\frac{3}{2}}(x - 3)^3, \quad x > 0.$$

a Show that
$$\mathrm{f}'(x) = k\,x^{\frac{1}{2}}(x - 1)(x - 3)^2,$$
where k is a constant to be found.

b Hence, find the coordinates of the stationary points of the curve $y = \mathrm{f}(x)$.

7
$$\mathrm{f}(x) = x\sqrt{2x+12}, \quad x \geq -6.$$

a Find $\mathrm{f}'(x)$ and show that $\mathrm{f}''(x) = \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$.

b Find the coordinates of the turning point of the curve $y = \mathrm{f}(x)$ and determine its nature.